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On the Optimality of Nonlinear Designs in the Control of Linear Decentralized Systems

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Abstract—**The purpose of this note is to point out the fact that in the decentralized optimum control of linear deterministic systems with instantaneous output feedback one might have nonlinear designs that are superior to the best linear ones.**

I. Introduction

Recently, interest has arisen in the control literature on the problem of optimal instantaneous feedback control of large-scale decentralized systems and especially for linear quadratic problems. That is, given the linear state dynamics

$$\dot{x}=Ax+\sum_{i=1}^{k}B_iu_i, \qquad x(0)=x_0$$

and the output equations

$$y_i=C_ix, \qquad i=1,\cdots,k$$

the problem is to determine the optimal control laws $\gamma_i(y_i)$, $i=1,\cdots,k$ so that with $u_i=\gamma_i(y_i)$ the cost functional

$$J=\int_0^T\left(x'Qx+\sum_{i=1}^{k}u_i'D_{ii}u_i\right)dt$$

attains its minimum within the class of admissible control laws. Here T can be taken to be finite or infinite depending on whether one is interested in a finite or infinite horizon problem.

In working on problems of this type it has been almost customary to assume an *a priori* probability distribution for the initial state vector x_0 (usually a covariance matrix is given) and to seek the optimal solution within the class of instantaneous linear feedback control laws, mainly because this assumption leads to structurally simpler and implementable solutions. The purpose of this note is to point out the fact that within the class of instantaneous feedback control laws an assumption of linearity might lead to results far from optimal since for the discrete-time version of the problem it is possible to produce examples which by resorting to Witsenhausen's famous counterexample [1] indicate superiority of nonlinear designs for such problems. We provide one such example in Section II.

Manuscript received April 23, 1976; revised June 3, 1976.
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II. Nonlinear Designs for a Discrete-Time Decentralized Control Problem

Consider the two-stage decentralized control problem with two-dimensional state dynamics

$$x(1)=x(0)+\begin{bmatrix}0\\1\end{bmatrix}u_0$$
$$x(2)=x(1)+\begin{bmatrix}0\\-1\end{bmatrix}u_1$$

and with scalar output equations

$$y_0=[0\ \ 1]x(0)$$
$$y_1=[1\ \ 1]x(1).$$

The objective is to determine the scalar control laws $u_0=\gamma_0(y_0), u_1=\gamma_1(y_1)$ such that

$$J=x'(2)\begin{bmatrix}0&0\\0&k^2\end{bmatrix}x(2)+u_0^2; \qquad k^2>0$$

is minimized. It is further assumed that the initial state vector $x(0)$ has mean zero and covariance $\begin{bmatrix}1&0\\0&\sigma^2\end{bmatrix}$.

Denoting the first component of $x(0)$ by v and the second component by x, J can now be written as

$$J=(x+u_0-u_1)^2k^2+u_0^2$$

and the output variables are

$$y_0=x$$
$$y_1=v+x+u_0.$$

The instantaneous feedback control problem has thus been converted into a nonclassical stochastic control problem of finding (Borel measurable) $\gamma_0(y_0)$ and $\gamma_1(y_1)$ such that the expected value of J is minimized. But this is precisely the counterexample of Witsenhausen in [1] where it has been shown that for some values of k^2 and σ^2 the nonlinear design

$$\gamma_0(x)=\sigma\,\mathrm{sgn}\,x-x$$
$$\gamma_1(y_1)=\sigma\tanh\sigma y_1$$

is much superior to the best linear one.

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A Simple and Direct Method of Reducing Order of Linear Systems Using Routh Approximations in the Frequency Domain

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Abstract—**A simple and direct method of approximating higher order systems by lower order ones based on Routh approximants without using reciprocal transformations is proposed. New algorithms for constructing reduced order transfer functions are presented. A method of approximating unstable systems with poles in RHP is also proposed resulting in unique transfer functions.**

Manuscript received April 23, 1976.
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