Noncooperative Carrier Sense Game in Wireless Networks

Kyung-Joon Park, Jennifer C. Hou, Fellow, IEEE, Tamer Başar, Fellow, IEEE, and Hwangnam Kim, Member, IEEE

Abstract—The performance of carrier sense multiple access (CSMA) wireless networks heavily depends on the level of spatial reuse, i.e., how many concurrent transmissions are allowed. Spatial reuse is primarily determined by physical carrier sense, and a key parameter for physical carrier sense is the carrier sense threshold. Our focus is on how to control the carrier sense threshold for improving network performance. We present a noncooperative game-theoretic framework, which leads to a fully distributed algorithm for tuning the carrier sense threshold. We introduce a utility function of each node, which is a nondecreasing concave function of the carrier sense threshold. A pricing function is further introduced to mitigate severe interference among nodes. The cost function is defined as the difference between the pricing and the utility functions. We prove that the noncooperative carrier sense game admits a unique Nash equilibrium (NE) under some technical conditions. We derive sufficient conditions that ensure the convergence of the synchronous and asynchronous update algorithms. Based on the analysis, we propose a fully distributed algorithm, entitled noncooperative carrier sense update algorithm (NCUA). Our simulation study indicates that NCUA outperforms standard CSMA with respect to the per-node throughput by 10–50%.

Index Terms—CSMA wireless network, spatial reuse, physical carrier sense, noncooperative game.

I. INTRODUCTION

MULTI-HOP wireless networks, such as wireless mesh networks, have emerged to be a promising cost-effective paradigm for the next-generation wireless technology [1]. In these networks, instead of using a pre-installed central entity for coordinating the radio channel, a distributed access mechanism is usually deployed to arbitrate access to the shared medium. One of the most widely used medium access mechanisms is carrier sense multiple access (CSMA).

A critical performance metric in wireless networks is network capacity, i.e., the average number of data bits that can be transported simultaneously in the network. In CSMA wireless networks, network capacity heavily depends on the level of spatial reuse, i.e., how many concurrent transmissions can be allowed in the network. In principle, spatial reuse cannot be arbitrarily exploited due to a tradeoff between spatial reuse and interference mitigation. As the level of spatial reuse is increased by allowing more concurrent transmissions, the fraction of successful transmissions decreases due to the increased interference among concurrent transmissions. Since the overall network capacity depends on the total number of successful concurrent transmissions, there exists a certain level of spatial reuse that maximizes the network capacity.

The level of spatial reuse is primarily characterized by physical carrier sense as follows: Before each transmission, a sender listens to the channel and determines whether or not the channel is busy by comparing the received signal strength with its carrier sense threshold. If the received signal strength is below the carrier sense threshold, the sender considers the channel to be idle and starts its transmission. Otherwise, the sender considers the channel to be busy and defers its transmission. Since the received signal strength is proportional to the transmit power of the corresponding transmitter, both the carrier sense threshold and the transmit power are major control knobs for exploiting the level of spatial reuse.

The issue of transmit power control for exploiting spatial reuse has been extensively studied in the context of topology control by graph-theoretic approaches, where the main objective is to reduce transmit power to mitigate MAC interference while preserving graph-theoretic network connectivity (a comprehensive survey on topology control can be found in [2]). Transmit power control has also been broadly studied from the viewpoint of maximizing network capacity (see [1] and the references therein). The common basic idea in these studies is to adopt an appropriate transmit power in order to exploit spatial reuse and increase the overall network capacity. In contrast, in spite of its importance, the issue of tuning the carrier sense threshold has not been paid much attention until recently, when a number of studies have been carried out [3]–[7]. Nevertheless, there have been few analytical studies on how to control the carrier sense threshold in a decentralized manner.

The aim of this paper is thus to investigate the problem of improving network capacity by carefully tuning the carrier sense threshold in CSMA wireless networks. In particular, we build an analytical framework and a distributed algorithm therein for tuning the carrier sense threshold. We first derive the collision probability of each node by taking into account the essential feature of physical carrier sense. Then, analogously as the power control problems in wireless networks [8], [9], a noncooperative game-theoretic framework naturally be-
comes a very efficient tool for distributed control of the carrier sense threshold. In order to develop reasonable utility and pricing functions, we identify the main consequences when a node increases its carrier sense threshold as follows: (i) The chance for channel access of the node will increase because it will care for fewer nodes. (ii) Both the interference from the node to others and that from others to it will become more severe. In order to characterize the first property, we introduce a nondecreasing concave function of each node’s carrier sense threshold as a utility function. Then, to balance out these conflicting factors, we further introduce a pricing function by incorporating the collision probability of each node. The overall cost function of each node is then defined as the difference between the pricing and the utility functions. A detailed explanation on the problem formulation will be given in Section III.

We prove that the noncooperative carrier sense game admits a unique Nash equilibrium (NE) for uniformly strictly concave utility functions under some technical assumptions. We also derive sufficient conditions that ensure the convergence of synchronous and asynchronous versions of a gradient-based algorithm for updating the carrier sense threshold. Based on the analysis, we propose a fully distributed algorithm, called noncooperative carrier sense update algorithm (NCUA), for tuning the carrier sense threshold. NCUA has the following desirable properties: (i) NCUA is fully distributed and does not require information exchange among nodes; (ii) NCUA is adaptive to network environment in its nature; (iii) By incorporating the collision probability into the cost function, NCUA significantly improves the overall network capacity compared to the standard CSMA. The simulation study indicates that NCUA outperforms standard CSMA with respect to the per-node throughput by 10–50%, when the latter operates over a wide range of the carrier sense threshold.

The rest of the paper is organized as follows. In Section II, we introduce the network propagation and interference models as well as the notions used in our analysis throughout the paper. Based on the model, we derive the conditional collision probability of each node as a function of carrier sense thresholds. Then, in Section III, we formulate the problem of updating the carrier sense threshold as a noncooperative carrier sense game. Following that, we derive in Section IV a sufficient condition for the existence of a unique NE, and show the convergence properties of synchronous and asynchronous algorithms. In Section V, based on the analysis, we propose NCUA and evaluate its performance via a simulation study. Finally, the paper ends with the concluding remarks in Section VI, and an Appendix.

II. NETWORK MODEL

The CSMA wireless network model used in our analysis consists of a set of $N$ nodes, denoted by $\mathcal{N} = \{1, \ldots, N\}$. For each node $i \in \mathcal{N}$, let $r(i) \in \mathcal{N}$ denote the corresponding receiver. Let $P_i$ and $P_{r(i)}$ denote the transmit power of node $i$ and the received power at $r(i)$, respectively. Then, $P_{r(i)}$ is expressed as $P_{r(i)} = G_{r(i),i}F_{r(i),i}P_i$, where $G_{r(i),i}$ and $F_{r(i),i}$ respectively represent the path loss and the Rayleigh fading from sender $i$ to receiver $r(i)$, which is a widely used model for wireless channel environments [10]. In addition, let $\tau_i$ denote the channel access probability of node $i$, which denotes the probability that node $i$ attempts to transmit a packet when it detects the channel idle. For further use, we also introduce $G_{r(i),j}$ and $F_{r(i),j}$ as the corresponding quantities effecting receiver $r(i)$ where the transmitter is sender $j$ (which contributes to interference at $r(i)$). As the magnitude of Rayleigh fading, $F_{r(i),j}$ is independent across nodes, and is exponentially distributed with unit mean. Here, we assume that the interference from other senders is much larger than the background noise in the receivers, and thus do not consider the noise in our analysis.

As a necessary condition for receiver $r(i)$ to correctly decode the symbols, $P_{r(i)}$ should be larger than or equal to the receive sensitivity of $r(i)$, denoted by $\beta(r(i)$, i.e.,

$$P_{r(i)} = G_{r(i),i}F_{r(i),i}P_i \geq \beta(r(i).$$

In addition, for successful reception at the receiver $r(i)$, the received power $P_{r(i)}$ should be large enough so that the interference from other nodes does not prevent the receiver from correctly decoding the symbols. This condition is usually characterized by the signal to interference ratio (SIR) as follows:

$$\text{SIR}_{r(i)} = \frac{P_{r(i)}}{I_{r(i)}} \geq \gamma(r(i).$$

where $I_{r(i)} = \sum_{j \neq i} G_{r(i),j}F_{r(i),j}P_j$ and $\gamma(r(i)$ is called the SIR threshold of the receiver $r(i)$.

The collision set of receiver $r(i)$, denoted by $C_{r(i)}$, is defined as a set of nodes whose simultaneous transmission with node $i$, taken one at a time, will prevent $r(i)$ from correctly decoding the symbols of node $i$ as follows:

$$C_{r(i)} = \{j \mid P_{r(i)} < \gamma(r(i)G_{r(i),j}F_{r(i),j}P_j\}.$$

Now, let $x_i$ denote the carrier sense threshold of node $i$. If the signal strength perceived at node $i$ is larger/smaller than $x_i$, the channel is considered busy/idle by node $i$. For a given node $i$, the carrier sense set $S_i(x_i)$ is defined as

$$S_i(x_i) = \{j \mid G_{i,j}F_{i,j}P_j \geq x_i\}.$$

Thus, node $i$ will be silenced if any node in $S_i(x_i)$ transmits. In a similar manner, let $L_i$ denote the silenced set of node $i$, which is defined as

$$L_i = \{j \mid G_{j,i}F_{j,i}P_j \geq x_i\}.$$

Hence, every node $j \in L_i$ will be silenced while node $i$ transmits. Note that the collision set $C_{r(i)}$ and the silenced set $L_i$ are independent of the carrier sense threshold $x_i$ by their definitions, while the carrier sense set $S_i(x_i)$ is a function of $x_i$, which satisfies $S_i(y) \subset S_i(x)$ if $x < y$ from its definition. In fact, what each node does by carrier sense is to detect the status of the channel before each transmission attempt. Hence, even when $C_{r(i)} \subset S_i(x_i)$, node $i$’s transmission may still fail if interrupted by a single simultaneous transmission in $C_{r(i)} \setminus L_i$ or multiple concurrent transmissions in $(\mathcal{N} \setminus C_{r(i)}) \setminus L_i$. The overall notation is summarized in Table I.

Now, we look into the collision probability of a node when it attempts to transmit. There are two sources of transmission collisions, namely, collision incurred by simultaneous
transmissions in $C_r(i)$ and collision incurred by multiple concurrent transmissions outside $C_r(i)$ that accumulate mediately violates the SIR constraint at the receiver $r(i)$. Let $q_i(x)$ denote the conditional collision probability of node $i$ given that a transmission attempt is made. Then, we have

$$q_i(x) = P[\text{Simultaneous tx in } C_r(i)] + P[\text{No simultaneous tx in } C_r(i)] \times P[\text{SIR}_{r(i)} < \gamma_r(i) | \text{No simultaneous tx in } C_r(i)]$$

$$= s_i + (1 - s_i) h_i(x),$$

where $s_i := P[\text{Simultaneous tx in } C_r(i)]$, $h_i(x) := P[\text{SIR}_{r(i)} < \gamma_r(i) | \text{No simultaneous tx in } C_r(i)]$, and $x$ is an abbreviation for transmission. Here, $h_i(x)$ can be further expressed as follows:

$$h_i(x) = P \left[ \frac{\sum_{k \in \Gamma_i(x)} P_{r(i)} G_{r(i), k} P_k F_k}{\sum_{k \in \Gamma_i(x)} G_{r(i), k} P_k F_k} < \gamma_r(i) \right],$$

where $\Gamma_i(x)$ denotes the set of nodes which concurrently transmit with node $i$ after no simultaneous transmission in $C_r(i)$ has occurred.

### III. Problem Formulation: Noncooperative Carrier Sense Game

We now present a noncooperative game-theoretic framework for decentralized control of physical carrier sense. First, let $U_i$ denote a utility function of node $i$, which the node seeks to maximize. Then, one possible approach is to formulate the problem of maximizing the network utility in a centralized fashion as

$$\max_{x \in X} \sum_{i=1}^{N} U_i(x),$$

where $x_i$ denotes the carrier sense threshold of node $i$ with $x_i \in X_i = [x_{min}, x_{max}], X = \prod_{i=1}^{N} X_i$, and $x = (x_1, \ldots, x_N)$. Note that (P1) is formulated from the system perspective, and hence the computation of its solution will naturally require a centralized algorithm. However, due to the lack of a pre-installed centralized infrastructure in CSMA wireless networks, it is more practical to formulate the problem from the perspective of individual nodes. We first consider the case where each node simply attempts to maximize its own utility function without caring for other nodes. This situation corresponds to the following optimization problem for each node $i$:

$$\max_{x_i \in X_i} U_i(x_i, x_{-i}), \ \forall i \in N,$$

where $x_{-i} = (x_1, \ldots, x_{i-1}, x_{i+1}, \ldots, x_N)$. Since the utility function $U_i$ is generally nondecreasing with respect to $x_i$ for given $x_{-i}$, (P2) admits the unique solution $x^* = [x_{max}, \ldots, x_{max}]$, which is trivially also the unique NE. In other words, if each node does not care for others, the best strategy for each node is to increase its carrier sense threshold as much as possible in order to increase its own utility. This selfish behavior results in an equilibrium of $x^* = [x_{max}, \ldots, x_{max}]$, which will give poor network performance because of significantly increased interference among the nodes.

This undesirable selfish behavior can be resolved by imposing a pricing function in (P2). In order to develop reasonable utility and pricing functions, we need to characterize the main effects of the carrier sense threshold on network performance. We identify the following main consequences as node $i$ increases its carrier sense threshold $x_i$: (i) The chance for channel access of node $i$ will be increased because node $i$ will care for fewer nodes and thus will sense an idle channel more frequently. (ii) In the meantime, the interference from node $i$ to others will be more severe because of more aggressive channel access of node $i$. Furthermore, the interference from other concurrent transmissions to node $i$ will be increased since the average number of sensed nodes is decreased.

By taking into account the first property, we introduce as a utility function a nondecreasing concave function of each node’s carrier sense threshold. Furthermore, to balance out these conflicting consequences of increasing the carrier sense threshold, we adopt a pricing function $P_i(x) := \int_{\xi}^{x_i} q_i(\xi, x_{-i}) - \eta_i \ d\xi$ by introducing a threshold $\eta_i$ on the collision probability for each node $i$. Since $q_i$ is increasing in $x_i$ for given $x_{-i}$, it can be easily verified that $P_i$ is convex in $x_i$ and the minimum is attained when $q_i = \eta_i$ for given $x_{-i}$. Thus, by comparing the collision probability $q_i$ with $\eta_i$, the pricing function attempts to maintain a reasonable carrier sense threshold in order to keep $q_i$ around $\eta_i$.

Now, we provide a game-theoretic framework with the introduced utility and pricing functions. First, node $i$’s overall cost function $J_i$ is defined as

$$J_i(x_i, x_{-i}) = P_i(x_i, x_{-i}) - U_i(x_i, x_{-i}).$$

A similar selfish behavior has been addressed in the power control problem of wireless networks in [11].

Note that heterogeneous values for $\eta_i$’s can further improve the overall network performance. An intuitive approach is that each node iteratively reduces $\eta_i$ in a large time scale as long as $q_i$ remains around $\eta_i$. However, a more rigorous analysis is required for this kind of algorithms. It will be an issue of future research to develop an efficient algorithm for tuning $\eta_i$ in an analytical framework.
With (4), we have the following NE computation.

\[
\min_{\xi \in \mathbb{X}} J_i(\xi, x_{-i}) = J_i(x_i, x_i), \quad \forall i \in \mathcal{N}. \quad (P3)
\]

One may devise a cooperative solution, which makes (P3) equivalent to (P1). In fact, (P3) will correspond to (P1) if \( P_i(x), \forall i \in \mathcal{N} \), were chosen such that

\[
\frac{\partial P_i(x)}{\partial x_i} = - \sum_{j \in \mathcal{N} \setminus \{i\}} \frac{\partial U_j(x)}{\partial x_i}, \quad (5)
\]

which can be obtained by equating \(-\partial J_i/\partial x_i\) to \( \sum_{j=1}^{N} \partial U_j/\partial x_i \). However, implementing a cooperative solution based on (5) requires a feedback mechanism for conveying all the information on \( \partial U_j/\partial x_i, \forall j \in \mathcal{N} \setminus \{i\} \). This information exchange will be an expensive overhead in wireless networks. Hence, we consider a noncooperative carrier sense game in order to develop a fully distributed control algorithm.

In our framework, \( U_i \) will depend only on \( x_i \), and thus be denoted as \( U_i(x_i) \). We further introduce the following two technical assumptions for \( U_i(x_i) \).

**A1.** \( U_i(x_i) \) is chosen to be twice continuously differentiable, nondecreasing and uniformly strictly concave in \( x_i \), i.e.,

\[
\frac{\partial U_i(x_i)}{\partial x_i} \geq 0, \quad \frac{\partial^2 U_i(x_i)}{\partial x_i^2} \leq -\eta < 0, \quad \forall x_i, i \in \mathcal{N}
\]

for some \( \eta > 0 \).

**A2.** In order to ensure an inner NE, \( U_i(x_i) \) is further chosen to satisfy

\[
\frac{\partial J_i}{\partial x_i} < 0 \text{ at } x_i = x_{\min}, \quad \frac{\partial J_i}{\partial x_i} > 0 \text{ at } x_i = x_{\max}, \quad \forall x_{-i}, i \in \mathcal{N}.
\]

In the following section, we derive conditions on the existence of a unique NE and the convergence of an iterative update algorithm for (P3).

**IV. NASH EQUILIBRIUM AND CONVERGENCE ANALYSIS**

**A. Existence and Uniqueness of Nash Equilibrium**

The first step for solving the noncooperative game of (P3) is to check the existence and the uniqueness of an equilibrium. Here, a Nash equilibrium (NE) is defined as a set of carrier sense thresholds, denoted by \( x^* = (x^*_1, \ldots, x^*_N) \), which satisfies the following property: no node can benefit by deviating from its carrier sense threshold while those of other nodes are kept fixed. In the mathematical sense, \( x^* \) is a NE if \( x^*_i, \forall i \in \mathcal{N} \), satisfies \( x^*_i = \arg\min_{x_i \in \mathcal{X}_i} J_i(x_i, x^*_{-i}) \).

We have the following result on the NE of (P3):

**Theorem 1** Under A1 and A2, (P3) admits a unique inner Nash equilibrium if

\[
\eta > \frac{L \tau_{\min}}{e(1 - \tau_{\max})x_{\min}},
\]

where \( L = \ln(1 + \gamma_{\max} P_{\max}/\beta_{\min}), \gamma_{\max} := \max_i \gamma_{\tau(i)}, \beta_{\min} := \min_i \beta_{\tau(i)}, \tau_{\max} := \max_i \tau_i \), and \( P_{\max} \) is the maximum transmit power.

**Proof:** See the appendix.

**B. Convergence of an Iterative Update Algorithm for Carrier Sense Thresholds**

Next, we investigate the convergence properties of synchronous and asynchronous update algorithms for the carrier sense threshold. Consider a discrete-time update scheme where each node updates its carrier sense threshold for solving (P3) as follows:

\[
x_i(t + 1) = x_i(t) - \alpha_i \frac{\partial J_i(x_i(t))}{\partial x_i}, \quad \forall i \in \mathcal{N}, \quad (6)
\]

where \( t = 1, 2, \ldots \) denotes the update time instants, and \( \alpha_i \) is the step size. Under the synchronous update scheme, all the nodes update their carrier sense thresholds simultaneously. Theorem 2 below provides a sufficient condition for the convergence of (6) in the synchronous case.

**Theorem 2** The synchronous update algorithm

\[
x_i(t + 1) = x_i(t) - \alpha_i \frac{\partial J_i(x_i(t))}{\partial x_i}, \quad \forall i \in \mathcal{N}
\]

converges to the unique NE, \( x^* := [x^*_1, \ldots, x^*_N] \), on \( X \) if, \( \forall x_i \in \mathcal{N} \),

\[
\alpha_i < \frac{e(1 - \tau_{\max})x_{\min}}{e(1 - \tau_{\max})x_{\min} u_{i,\max} + L(N - 1)\tau_{\max}} \quad \text{and} \quad \eta > \frac{L(N - 1)\tau_{\max}}{e x_{\min}},
\]

where \( u_{i,\max} := -\partial^2 U_i(x_{\min})/\partial x_i^2 \), and \( L, \tau_{\max} \) are defined in the same way as in Theorem 1.

**Proof:** The proof follows lines similar to those in [12, Proposition 1.11 p. 194]. First, we derive a sufficient condition on the step size \( \alpha_i \). Let \( f_i(x) := \partial J_i(x)/\partial x_i \). Then, for the convergence of (6), the step size \( \alpha_i \) should satisfy \( 0 < \alpha_i < 1/K \) where \( K \) is a positive constant such that \( \partial f_i/\partial x_i \leq K, \forall x \in X \). From (2) and (3), we have

\[
\frac{\partial f_i(x)}{\partial x_i} = \frac{\partial^2 P_i(x)}{\partial x_i^2} - \frac{\partial U_i(x_i)}{\partial x_i} \leq u_{i,\max} + \frac{\partial h_i(x)}{\partial x_i}, \quad (7)
\]

where \( u_{i,\max} := -\partial^2 U_i(x_{\min})/\partial x_i^2 \). From the appendix, we have

\[
\frac{\partial h_i(x)}{\partial x_i} \leq L \sum_{k \in \mathcal{N} \setminus \{i\}} \frac{\partial P[k \in \Gamma_i(x)]}{\partial x_i} \leq \frac{L(N - 1)\tau_{\max}}{e(1 - \tau_{\max})x_{\min}}. \quad (8)
\]

From (7) and (8), the step size \( \alpha_i \) should satisfy

\[
0 < \alpha_i < \frac{e(1 - \tau_{\max})x_{\min}}{e(1 - \tau_{\max})x_{\min} u_{i,\max} + L(N - 1)\tau_{\max}}.
\]

In a similar manner, from (17) and (18), we have

\[
\frac{\sum_{j \in \mathcal{N} \setminus \{i\}} \frac{\partial h_j(x)}{\partial x_j}}{\partial x_j} \leq L \sum_{j \in \mathcal{N} \setminus \{i\}} \frac{\partial P[j \in \Gamma_i(x)]}{\partial x_j} \leq \frac{L(N - 1)\tau_{\max}}{e x_{\min}}, \quad (9)
\]

and from (9), the condition \( \partial f_i/\partial x_i \leq \sum_{j \neq i} |\partial f_i/\partial x_j| \) in [12, Proposition 1.11 p. 194] is satisfied if \( \eta > L(N - 1)\tau_{\max}/(e x_{\min}) \).
A natural generalization of the synchronous algorithm is the asynchronous algorithm where only a random subset of nodes update their carrier sense thresholds at a given time instant. This is more realistic since it is difficult for nodes to synchronize their update instants in a practical setting. In fact, the convergence analysis in Theorem 2 directly applies to the asynchronous case, and we have the following result for the convergence in the asynchronous case.

**Theorem 3** Let \( U(k) \) denote a random subset of nodes updating their carrier sense threshold at time \( k \). Then, the asynchronous update algorithm

\[
x_i(t + 1) = \begin{cases} x_i(t) - \alpha_i \frac{\partial J}{\partial x_i}, & \text{if } i \in U(t + 1); \\ x_i(t), & \text{otherwise,} \end{cases}
\]

converges to the unique NE on \( X \) if, \( \forall i \in \mathcal{N} \),

\[
\alpha_i < \frac{e(1 - \tau_{\text{max}}) x_{\text{min}}}{e x_{\text{min}} + L(N - 1) \tau_{\text{max}}},
\]

\[
\eta > \frac{\sum_i u_i}{e x_{\text{min}}},
\]

where \( u_{i,\text{max}}, L, \) and \( \tau_{\text{max}} \) are defined in the same way as in Theorem 2.

**Proof:** First, we define a sequence of nonempty, convex, and compact sets

\[
X(k) := \left\{ x^1 - \delta(k), x^1 + \delta(k) \right\} \times \left\{ x^2 - \delta(k), x^2 + \delta(k) \right\} \times \cdots \times \left\{ x^N - \delta(k), x^N + \delta(k) \right\},
\]

where \( \delta(k) := \| x(k) - x^* \| \). From Theorem 2, \( \delta(k + 1) < \delta(k) \). Hence, \( \cdots \subset X(k + 1) \subset X(k) \subset \cdots \subset X \). Now recall two well-known sufficient conditions for the asynchronous convergence of a nonlinear iterative mapping \( x(t + 1) = T(x(t)) \) [12, p. 431].

(Synchronous Convergence Condition) We have \( T(x) \in X(k + 1), \forall k \) and \( x \in X(k) \). Furthermore, if \( \{ y^k \} \) is a sequence such that \( y^k \in X(k) \) for every \( k \), then every limit point of \( \{ y^k \} \) is a fixed point of \( T \).

(Box Condition) Given a closed and bounded set \( X_i \) in \( \mathbb{R} \), for every \( k \), there exist sets \( X_i(k) \subset X_i \) such that

\[
X(k) = X_1(k) \times X_2(k) \times \cdots \times X_N(k).
\]

Here, \( X_i \) is defined as \( [x_{i,\text{min}}, x_{i,\text{max}}] \), and \( X_i(k) := [x_i^* - \delta(k), x_i^* + \delta(k)] \). Thus, the box condition is satisfied by the definition of \( X_i \). Also, \( \delta(k) \) is monotonically decreasing in \( k \) by Theorem 2. Consequently, the convergence of the asynchronous update algorithm of (10) immediately follows from [12, p. 431].

**V. SIMULATION STUDY**

A. Noncooperative Carrier Sense Update Algorithm (NCUA)

Based on the analysis in Section III and Section IV, we propose a fully distributed algorithm, called noncooperative carrier sense update algorithm (NCUA), and evaluate its performance via J-Sim simulation [13]. The pseudo code of NCUA is given in Algorithm 1. In the initialization procedure of Algorithm 1, the initial value for \( x_i \) is given. Then, in the update procedure, the carrier sense threshold \( x_i \) is updated every \( \Delta \) seconds. First, the collision probability is estimated by counting the total number of unsuccessful transmissions during the interval of \( \Delta \). Then, \( x_i \) is updated based on (6) with \( U_i(x_i) = u_i \ln x_i \). In Algorithm 1, \( x_i \) and \( T_i \) are in pW and b/s, respectively.

There are a few points worth mentioning. First, NCUA is fully distributed and does not require information exchange among nodes. Second, in order to implement NCUA, it was required to choose a specific utility function \( U_i(x_i) \) in (6). Here, we adopt a logarithmic utility function parameterized by \( u_i \) for node \( i \) as above. The choice of other structures is a subject of future work.

**B. Simulation Setup**

In the simulation study, we use IEEE 802.11a as the MAC protocol. Table II gives the default values for the parameters used in the simulation. In particular, the receive sensitivity \( \gamma \) is set to the default value in IEEE 802.11a specification. Also, to precisely quantify the impact of adjusting carrier sense thresholds on the throughput performance, we disable the binary exponential back-off (BEB) mechanism in IEEE

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Definition</th>
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<tr>
<td>( N )</td>
<td>100</td>
<td>Number of nodes</td>
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<td>( P_{\text{len}} )</td>
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<td>(18, 54) Mb/s</td>
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<td>( CW )</td>
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<td>Minimal transmission range</td>
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<td>( u )</td>
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<td>Utility parameter in NCUA</td>
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<td>( 10^{-12} )</td>
<td>Update step size of NCUA</td>
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<td>( x_i^{\text{init}} )</td>
<td>-75 dBm</td>
<td>Initial value of ( x_i ) in NCUA</td>
</tr>
<tr>
<td>( \eta_i )</td>
<td>0.2</td>
<td>Collision probability threshold</td>
</tr>
</tbody>
</table>
standard CSMA over the entire range for both the data rates. For NCUA, every node sets the initial value of its carrier sense threshold. As shown in Fig. 1. The network topology used in the simulation study.

Figure 2 gives the average per-node throughput (averaged over all the source nodes) versus the carrier sense threshold for 802.11 DCF in order to decouple its effect. Instead, we use a fixed contention window size of 63 as given in Table II. Finally, the same value of $u_i$, denoted by $u$, is used for all the nodes.

To calculate the sufficient condition in Theorem 2, from Table II, we have $\tau_{\text{max}} = 2/(CW + 1) = 1/32$, $x_{\text{min}} = -84$ dBm, and $x_{\text{max}} = -73$ dBm. In addition, from the logarithmic utility function of $U_i(x_i) = u_i \log x_i$, we have $\eta = \min_i u_i/x_{\text{max}}^2$. With these values, the convergence condition in Theorem 2 gives $\alpha_i < 1.04 \cdot 10^{-12}$ and $u_i > 1.5 \cdot 10^{-11}$. Since these values are quite sufficient rather than necessary, we set $\alpha_i = 10^{-12}$ and $u_i = 10^{-11}$ as the default values in our simulation study. Note that we also perform simulation with different values of $u_i$ to verify its effect on network performance.

Figure 1 shows the network topology used for the simulation study. A total of $N = 100$ nodes are distributed in an area of 500$\times$500 m$^2$. First, half of the nodes are randomly located as senders. Then, for each sender, a receiver is randomly located inside a circle with center at the sender and radius $d_{\text{min}}$.

As already explained, the transmit power used by each node is randomly chosen to give a transmission range from $d_{\text{min}}$ up to $d_{\text{max}}$. All sources generate CBR traffic at their full data rates. For NCUA, every node sets the initial value of its carrier sense threshold to $-75$ dBm.

C. Simulation Results

Figure 2 gives the average per-node throughput (averaged over all the source nodes) versus the carrier sense threshold for $R = 18$ Mb/s and 54 Mb/s, respectively. Note that the $x$-axis in Fig. 2 only applies to the standard CSMA because NCUA dynamically updates the carrier sense threshold. As shown in Fig. 2, NCUA achieves better throughput performance than standard CSMA over the entire range for both the data rates of 18 and 54 Mb/s. Specifically, NCUA achieves 11.76% and 16.13% higher throughput than the maximal throughput achieved by the standard CSMA at $R = 18$ Mb/s and 54 Mb/s, respectively. It should be noted that, in practice, it is extremely difficult to set an optimal carrier sense threshold under the standard CSMA. Furthermore, as shown in Fig. 2, if an inappropriate carrier sense threshold is chosen, standard CSMA gives inferior throughput performance. Consequently, the performance of NCSU will be much better than that of the standard CSMA in practice. For example, about 40% and 50% for $R = 18$ Mb/s and $R = 54$ Mb/s when the carrier sense threshold is set to $-72$ dBm for standard CSMA.

Figure 3 gives the temporal behavior of how the carrier sense threshold of each node adapts under NCUA when $R = 54$ Mb/s. In particular, it shows the time traces of the carrier sense thresholds of four randomly selected nodes. Note that the traces of the carrier sense thresholds for other nodes exhibit similar trends and hence are not shown. As given in Fig. 3, the carrier sense threshold of each node initially starts from the default initial value of $-75$ dBm and becomes stabilized.

Figure 4 depicts the relation between the throughput performance and the utility parameter $u$. Although the sufficient condition derived from our analysis provides a guideline for setting the utility parameter $u$ in terms of the convergence, it is worth to look into the actual performance with respect to $u$. The utility parameter $u$ can be interpreted as the incentive for increasing the carrier sense threshold. Hence, if $u$ is too small, the carrier sense threshold is set to an inappropriately small value, which may result in degradation of the throughput performance. However, we can verify from Fig. 4 that the performance of NCUA is quite robust to changes in $u$ over a wide range. Note that it is very difficult, though not impossible, to derive an explicit relation between the network performance and the utility parameter $u$ because of the complex problem...
Finally, we evaluate the throughput sensitivity with respect to the number of nodes in the network. Figure 5 shows the average node throughput of NCUA and standard CSMA with respect to the number of nodes. Overall, we can verify from Fig. 5 that NCUA performs better than standard CSMA in a wide range of the number of nodes. When the number of nodes is small, it is quite intuitive that the throughput performance is not very sensitive to the carrier sense threshold. This relation explains the small gap between NCUA and standard CSMA when the number of nodes is small in Fig. 5. It should be noted that the carrier sense threshold of standard CSMA in Fig. 5 is set to −75 dBm, which is quite a favorable value as shown in Fig. 2. If a less favorable value is chosen (for example, −80 dBm), the throughput gap between NCUA and standard CSMA will become larger.

VI. CONCLUSION AND FUTURE WORK

We have presented a noncooperative game-theoretic framework for distributed control of the carrier sense threshold in CSMA wireless networks. We have shown that the noncooperative carrier sense game admits a unique NE for uniformly strictly concave utility functions under some technical conditions. We have also derived sufficient conditions that ensure the convergence of both the synchronous and asynchronous versions of a gradient update algorithm for the carrier sense threshold. Our analysis has led to a fully distributed algorithm, called noncooperative carrier sense update algorithm (NCUA). The simulation study has shown that NCUA outperforms the standard CSMA with respect to the per-node throughput by 10–50%, when the latter operates over a wide range of the carrier sense threshold.

In our analysis, we have not considered the effect of the dynamic contention resolution algorithm, such as the binary exponential back-off (BEB) algorithm used in IEEE 802.11 DCF. Since the BEB algorithm has recently been shown to maximize a certain utility function in a noncooperative manner [14], it will be an interesting research avenue to study the interaction between physical carrier sense and contention resolution in a game-theoretic framework. In addition, since the operating point will be different for each node, heterogeneous values for \( q_i \) will further improve the overall network performance. Hence, it will be a subject of future work to devise a distributed algorithm for adjusting \( q_i \) of each node.

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APPENDIX

The proof follows a line of arguments similar to those in [9], [15]. First, we show the existence of a NE. The set of feasible carrier sense thresholds is defined as \( X = \prod_{i=1}^{N} X_i \), where \( x_i \in X_i = [x_{min}, x_{max}], \forall i \in \mathcal{N} \). Then, \( X \) is closed and bounded, and thus compact. Also, \( X \) is convex and has a nonempty interior. Furthermore, the cost function \( J_i(x) \) in (4) is convex in \( x_i \) under the condition (12), which is to be shown

3The simulation result for other values of the carrier sense threshold has been omitted due to the limit on the number of figures.
as part of the uniqueness proof below. Thus, by a standard theorem in game theory (for example, [16, Theorem 4.4, p. 176]) together with A2, there exists an inner NE.

Now, we consider the uniqueness of the NE. Let \( A_{ij} := \partial^2 J_i(x)/\partial x_i \partial x_j \) and \( B_i := \partial^2 J_i(x)/\partial x_i^2 \). Then, from the arguments used in [9], [15], the condition \( B_i > |A_{ij}|, \forall i, j \neq i \) is sufficient to show the uniqueness of NE. Hence, we derive a sufficient condition, under which \( B_i > |A_{ij}| \) is satisfied. Since the utility function \( U_i(x_i) \) is convex in \( x_i \), we have

\[
B_i = \frac{\partial^2 P_i(x)}{\partial x_i^2} - \frac{\partial^2 U_i(x_i)}{\partial x_i^2} \geq \eta. \tag{11}
\]

From (11), a sufficient condition for \( B_i > |A_{ij}| \) is given as follows:

\[
\eta > \left| \frac{\partial^2 P_i(x)}{\partial x_i \partial x_j} \right|. \tag{12}
\]

Thus, what remains to be done is to obtain an explicit expression for \( \partial^2 P_i(x)/\partial x_i \partial x_j \) in terms of \( x \). From the definition of the pricing function together with (2), we have

\[
\frac{\partial^2 P_i(x)}{\partial x_i \partial x_j} = \frac{\partial h_i(x)}{\partial x_j} = (1 - s_i)\frac{\partial h_i(x)}{\partial x_j}. \tag{13}
\]

By incorporating (14) into (3), \( h_i(x) \) in (13) becomes

\[
h_i(x) = E \left[ 1 - \prod_{k \in \Gamma_i(x)} \frac{1}{1 + \gamma_{r(i)} G_{r(i),k} F_k} \right], \tag{15}
\]

\[
\approx E \left[ \sum_{k \in \Gamma_i(x)} \ln \left( 1 + \frac{\gamma_{r(i)} G_{r(i),k} F_k}{G_{r(i),k} P_k} \right) \right], \tag{16}
\]

where the approximation in (16) comes from \( x \approx -\ln(1-x) \) for \( x \ll 1 \). Let the indicator function \( I_{\Gamma_i(x)}(k) \) be defined as

\[
I_{\Gamma_i(x)}(k) = \begin{cases} 1, & \text{if } k \in \Gamma_i(x) \; ; \\ 0, & \text{otherwise}. \end{cases}
\]

Then,

\[
h_i(x) = E \left[ \sum_{k \in \mathcal{N} \setminus \{i\}} \ln \left( 1 + \frac{\gamma_{r(i)} G_{r(i),k} P_k}{G_{r(i),k} P_k} \right) P [k \in \Gamma_i(x)] \right]. \tag{17}
\]

Now, we obtain an expression for \( P[k \in \Gamma_i(x)] \) in (17). In order for any node to interfere with node \( i \)'s transmission, either its transmission should not have been sensed by node \( i \) or it should not sense node \( i \)'s transmission and attempts to transmit. Thus,

\[
P[k \in \Gamma_i(x)] = \frac{\tau_k P_{SC} + (1 - \tau_k) P_{EC}}{\tau_k P_{SC} + (1 - \tau_k)}, \tag{18}
\]

where \( P_{SC} := P \left[ k \in \mathcal{N} \setminus S_i \right] = P[G_{r(i),k} F_k \leq x_i] = 1 - e^{-x_i/(G_{r(i),k} P_k)} \) and \( P_{EC} := P \left[ k \in \mathcal{N} \setminus L_i \right] = P[G_{r(k),k} F_k \leq x_k] = 1 - e^{-x_k/(G_{r(k),k} P_k)} \). In the meantime, from (1), we have

![Image](image1.png)

![Image](image2.png)

Fig. 5. Average node throughput versus the number of nodes in the network when the carrier sense threshold of standard CSMA is set to \(-75 \text{ dBm}\).
The condition for the uniqueness of NE is given as

\[ G_{r(i),k} P_k / (G_{r(i),i} P_i) \leq P_{max} / \beta_{r(i)} \],

where \( P_{max} \) is the maximum transmit power. Thus, by collecting (12), (13), (17), and (18) with some algebraic manipulation, the sufficient condition for the uniqueness of NE is given as

\[ \eta > \ln \left( \frac{1 + \gamma_{max} P_{max}}{\beta_{min}} \right) \cdot \frac{\tau_{max}}{\epsilon (1 - \tau_{max}) \beta_{min}}, \]

where \( \gamma_{max} := \max_i \gamma_{r(i)}, \beta_{min} := \min_i \beta_{r(i)}, \) and \( \tau_{max} := \max_i \tau_i. \)

**REFERENCES**


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